

**OPTION-B**

Paper : MAT-HE-5026

**(Mechanics)**

Full Marks : 80

Time : Three hours

**The figures in the margin indicate full marks for the questions.**

1. Answer the following questions :  $1 \times 10 = 10$

- (i) What is the physical significance of the moment of a force ?
- (ii) State Newton's second law of motion.
- (iii) Define angle of friction.
- (iv) Define the centre of gravity of a body.
- (v) What do you mean by terminal velocity ?
- (vi) What is the geometrical representation of the simple harmonic motion ?
- (vii) What is the length of arm of a couple equivalent to the couple  $(P, p)$  having constituent force of magnitude  $F$  ?

(viii) Can a force and a couple in the same plane be equivalent to a single force ?

(ix) Define a couple.

(x) State Hooke's law.

2. Answer the following questions :  $2 \times 5 = 10$

- (a) Find the greatest and least resultant of two forces acting at a point whose magnitudes are  $P$  and  $Q$  respectively.
- (b) Find the centre of gravity of an arc of a plane curve  $y = f(x)$ .
- (c) State the laws of static friction.
- (d) Show that impulse of a force is equal to the momentum generated by the force in the given time.
- (e) Write the expression for the component of velocity and acceleration along radial and cross-radial direction for a motion of a particle in a plane curve.

3. Answer **any four** questions of the following :  
5×4=20

(a) The line of action of a force  $F$  divides the angle between its component forces  $P$  and  $Q$  in the ratio 1 : 2. Prove that  $Q(F + Q) = P^2$ .

(b)  $P$  and  $Q$  are two like parallel forces. If  $P$  is moved parallel to itself through a distance  $x$ , show that the resultant of  $P$  and  $Q$  moves through a distance  $\frac{Px}{P+Q}$ .

(c)  $R$  is the resultant of two forces  $P$  and  $Q$  acting at a point and at a given angle. If the force  $P$  be doubled, show that the new resultant will be of magnitude  $\sqrt{2(P^2 + R^2)} - Q^2$ .

(d) A particle of mass  $m$  moves in a straight line under acceleration  $mn^2x$  towards a point  $O$  on the line, where  $x$  is the distance from  $O$ . Show that if  $x = a$  and  $\frac{dx}{dt} = u$  when  $t = 0$ , then at time  $t$ ,  
$$x = a \cos nt + \frac{u}{n} \sin nt.$$

(e) A particle moving with simple harmonic motion in a straight line has velocity  $v_1$  and  $v_2$  at distance  $x_1, x_2$  from the centre of its path. Show that if  $T$  be the period of its motion then  $T = 2\pi \sqrt{\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}}$ .

(f) Show that the sum of the kinetic energy and potential energy is constant throughout the motion when a particle of mass  $m$  falls from rest at a height  $h$  above ground.

4. Answer **any four** questions of the following :  
10×4=40

(a) Forces  $P, Q$  and  $R$  act along the sides  $BC, CA$  and  $AB$  of a triangle  $ABC$  and forces  $P', Q'$  and  $R'$  act along  $OA, OB$  and  $OC$ , where  $O$  is the centre of the circumscribed circle, prove that

(i)  $P \cos A + Q \cos B + R \cos C = 0$

(ii)  $\frac{PP'}{a} + \frac{QQ'}{b} + \frac{RR'}{c} = 0$

- (b) State and prove Lami's theorem. Forces  $P$ ,  $Q$  and  $R$  acting along  $OA$ ,  $OB$  and  $OC$ , where  $O$  is the circumcentre of triangle  $ABC$ , are in equilibrium. Show that

$$\frac{P}{a^2(b^2+c^2-a^2)} = \frac{Q}{b^2(c^2+a^2-b^2)} = \frac{R}{c^2(a^2+b^2-c^2)}$$

- (c) (i) Find the centre of gravity of a circular arc of radius  $a$  which subtends an angle  $2\alpha$  at the centre.
- (ii) Find the centre of gravity of a uniform parabolic area cut off by a double ordinate at a distance  $h$  from the vertex.
- (d) (i) Show that the least force which will move a weight  $W$  along a rough horizontal plane is  $W \sin \phi$ , where  $\phi$  is the angle of friction.
- (ii) If a body is placed upon a rough inclined plane, and is on the point of sliding down the plane under the action of its weight and the reactions of the plane only, show that the angle of inclination of the plane to the horizon is equal to the angle of friction.

- (e) A particle moves in a straight line under an attraction towards a fixed point on the line varying inversely as the square of the distance from the fixed point. Investigate the motion.
- (f) A particle moves in a straight line  $OA$  starting from the rest at  $A$  and moving with an acceleration which is directed towards  $O$  and varies as the distance from  $O$ . Discuss the motion of the particle. Hence define simple harmonic motion and time period of the motion.
- (g) Find the component of acceleration of a point moving in a plane curve along the initial line and the radius vector. Also find the component of acceleration perpendicular to initial line and perpendicular to radius vector.
- (h) A particle is falling under gravity in a medium whose resistance varies as the velocity. Find the distance and velocity at any time  $t$ . Also find the terminal velocity of the particle.